Tenseur diélectrique : corrections thermiques

(f_α de distribution maxwellienne, notations Ichimaru p 89)

$$\bar{\bar{\varepsilon}}(\bar{k},\omega) = \bar{\bar{1}} - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2} \left\{ \sum_{n=-\infty}^{+\infty} \frac{Z_0^\alpha}{Z_n^\alpha} \bar{\bar{\Pi}}(\beta_\alpha, Z_n^\alpha; n) \left[1 - W_I(Z_n^\alpha) \right] - Z_0^2 \bar{z}\bar{z} \right\}$$

avec

$$\bar{\bar{\Pi}} = \begin{bmatrix} \frac{n^2}{\beta_\alpha}\Lambda_n(\beta_\alpha) & ; \; in\Lambda_n'(\beta_\alpha) & ; \; sg(k_\parallel)\frac{n}{\sqrt{\beta_\alpha}} Z_n^\alpha \Lambda_n(\beta_\alpha) \\ -in\Lambda_n'(\beta_\alpha) & ; \; \frac{n^2}{\beta_\alpha}\Lambda_n(\beta_\alpha) - 2\beta_\alpha\Lambda_n'(\beta_\alpha) & ; \; -i\, sg(k_\parallel)\sqrt{\beta_\alpha} Z_n^\alpha \Lambda_n'(\beta_\alpha) \\ sg(k_\parallel)\frac{n}{\sqrt{\beta_\alpha}} Z_n^\alpha \Lambda_n(\beta_\alpha) & ; \; i\, sg(k_\parallel)\sqrt{\beta_\alpha} Z_n^\alpha \Lambda_n'(\beta_\alpha) & ; \; Z_n^{\alpha 2}\Lambda_n(\beta_\alpha) \end{bmatrix}$$

$$Z_n^\alpha = \frac{\omega - n\Omega_{c\alpha}}{|k_\parallel| \left(T_\alpha / M_\alpha \right)^{\frac{1}{2}}} \quad ; \quad \Omega_{c\alpha} = \frac{+q_\alpha B_0}{M_\alpha} \quad ; \quad \begin{cases} \bar{z} \parallel \bar{B}_0 \\ \bar{x} \parallel \bar{k}_\perp \end{cases}$$

$$\beta_\alpha = k_\perp^2 \left(\frac{T_\alpha}{M_\alpha}\right) \Big/ \Omega_\alpha^2 = k_\perp^2 \rho_\alpha^2 \quad ; \quad v_{th\alpha} = \sqrt{\frac{T_\alpha}{M_\alpha}} = \langle v_\parallel^2 \rangle^{\frac{1}{2}} = \frac{1}{\sqrt{2}} \langle v_\perp^2 \rangle^{\frac{1}{2}}$$

$$\Lambda_n(\beta) = I_n(\beta)\, e^{-\beta}$$

$$W_I(z) = \frac{1}{\sqrt{2\pi}} \int_C \frac{x}{x-z} \exp\left(-\frac{x^2}{2}\right) dx = 1 + \frac{z}{\sqrt{2}} Z\left(\frac{z}{\sqrt{2}}\right) = -\frac{1}{2} Z'\left(\frac{z}{\sqrt{2}}\right)$$

• $z \gg 1 \qquad W_I(z) \sim -\frac{1}{z^2} - \frac{3}{z^4} \dots + i\sigma\sqrt{\frac{\pi}{2}}\, z \exp\left(-\frac{z^2}{2}\right)$

$$\sigma = \begin{cases} 0 & \text{si } \operatorname{Im} z > 0 \\ 1 & \text{si } \text{—} = 0 \\ 2 & \text{si } \text{—} < 0 \end{cases}$$

$z \ll 1 \qquad W_I(z) \sim 1 - z^2 + \frac{z^4}{3} \dots + i\sqrt{\frac{\pi}{2}}\, z \exp\left(-\frac{z^2}{2}\right)$

• $\Lambda_{-n}(\beta) = \Lambda_{+n}(\beta)$, développement en serie :

$$\Lambda_0(\beta) = 1 - \beta + \frac{3\beta^2}{4} - \frac{5\beta^3}{12} + \frac{35\beta^4}{192} + \dots$$

$$\Lambda_1(\beta) = \frac{\beta}{2} - \frac{\beta^2}{2} + \frac{5\beta^3}{16} - \frac{7\beta^4}{48} + \dots$$

$$\Lambda_2(\beta) = \frac{\beta^2}{8} - \frac{\beta^3}{8} + \frac{7\beta^4}{96} + \dots$$

$$\Lambda_3(\beta) = \frac{\beta^3}{48} - \frac{\beta^4}{48} + \dots$$

$$\Lambda_4(\beta) = \frac{\beta^4}{384} + \dots$$

• $\bar{\bar{\varepsilon}} = \bar{\bar{1}} + \frac{i}{\varepsilon_0 \omega}\bar{\bar{\sigma}} \qquad \left(e^{i(\bar{k}\cdot\bar{r} - \omega t)}\right)$

Développement au 1er ordre en β_α

1) ε^α_{xx}

$n = 0$ terme d'ordre -1 nul
$n = -1, 1$ termes d'ordre 0
$n = -2, 2$ termes d'ordre 1

$$A_n = \frac{\omega_{p\alpha}^2}{\omega^2}\frac{Z_0^\alpha}{Z_n^\alpha}$$

$$\varepsilon_{xx} = (1) - \left(\frac{1}{2} - \frac{\beta}{2}\right)\left[A_1(1 - W[Z_1]) + A_{-1}(1 - W[Z_{-1}])\right] - \frac{\beta}{2}\left[A_2(1 - W[Z_2]) + A_{-2}(1 - W[Z_{-2}])\right]$$

$$\boxed{\begin{aligned}\varepsilon_{xx} = (1) &- \left(\frac{1}{2} - \frac{\beta}{2}\right)\left[\frac{2\omega_p^2}{\omega^2 - \Omega_c^2} - \frac{\omega_p^2}{\omega}\left(\frac{W[Z_1]}{\omega - \Omega_c} + \frac{W[Z_{-1}]}{\omega + \Omega_c}\right)\right] \\ &- \frac{\beta}{2}\left[\frac{2\omega_p^2}{\omega^2 - 4\Omega_c^2} - \frac{\omega_p^2}{\omega}\left(\frac{W[Z_2]}{\omega - 2\Omega_c} + \frac{W[Z_{-2}]}{\omega + 2\Omega_c}\right)\right]\end{aligned}}$$

• Approximation basse fréquence $\omega \ll \Omega_c$ et faible décalage Doppler parallèle $|k_\parallel| v_{th} \ll \Omega_c$ ($\Rightarrow Z_{n\neq 0} \gg 1$ mais Z_0 peut être $\lesssim 1$) :

$$\boxed{\varepsilon^\alpha_{xx} = (1) + \frac{\omega_{p\alpha}^2}{\Omega_{c\alpha}^2}\left(1 - \frac{3}{4}\beta_\alpha\right)}$$

$$\hookrightarrow \boxed{\langle v_x \rangle_\alpha = \frac{m_\alpha}{q_\alpha B_0^2}\left(1 + \frac{3}{4}\rho_\alpha^2 \frac{\partial^2}{\partial x^2}\right)\frac{\partial E_x}{\partial t}}$$

(dérive de polarisat°)

- $\omega \ll \Omega_c$, $|k_\parallel| v_{th} \ll \Omega_c$ mais β quelconque :
approximation basse fréquence, pas de particule resonnante par effet Doppler,
mais pas d'approximation sur le rayon de Larmor !

$$\varepsilon_{xx} = (1) - \omega_p^2 \frac{1}{\omega} \sum_{n=1}^{+\infty} \left(\frac{1}{\omega - n\Omega_c} + \frac{1}{\omega + n\Omega_c} \right) \frac{n^2}{\beta} \Lambda_n(\beta) \left[1 - \cancel{W(z_{\pm n})} \right]$$

$$\varepsilon_{xx} = (1) + \frac{2\omega_p^2}{\Omega_c^2} \sum_{n=1}^{+\infty} \frac{1}{\beta} \Lambda_n(\beta) \qquad , \sum_{-\infty}^{+\infty} \Lambda_n(\beta) = 1$$

$$\boxed{\varepsilon_{xx}^\alpha = (1) + \frac{\omega_{p\alpha}^2}{\Omega_{c\alpha}^2} \frac{1 - \Lambda_0(\beta_\alpha)}{\beta_\alpha}}$$

$\Rightarrow$ possibilité de traiter la dérive de polarisation avec de grands rayons de Larmor ...

2) ε_{yy}^α

$n = 0$ terme d'ordre 1
$n = -1, 1$ termes d'ordre 0 et 1
$n = -2, 2$ termes d'ordre 1 et 2

(terme de gauche identique à ε_{xx})

$$\varepsilon_{yy} = \varepsilon_{xx} + 2\beta(-1) A_0 (1 - W[z_0]) + 2\beta\left(\frac{1}{2}\right) \left[A_1 (1 - W[z_1]) + A_{-1} (1 - W[z_{-1}]) \right]$$

$$\boxed{\begin{aligned} \varepsilon_{yy} = \varepsilon_{xx} &- 2\beta \frac{\omega_p^2}{\omega^2} \left(1 - W\left[\frac{\omega}{|k_\parallel| v_{th}}\right] \right) + \beta \frac{2\omega_p^2}{\omega^2 - \Omega_c^2} \\ &- \beta \frac{\omega_p^2}{\omega} \left(\frac{W[z_1]}{\omega - \Omega_c} + \frac{W[z_{-1}]}{\omega + \Omega_c} \right) \end{aligned}}$$

- $\omega \ll \Omega_c$ et $|k_\parallel| v_{th} \ll \Omega_c$:

$$\boxed{\varepsilon_{yy}^\alpha = (1) + \frac{\omega_{p\alpha}^2}{\Omega_{c\alpha}^2} \left(1 - \frac{11}{4}\beta_\alpha - 2\beta_\alpha \frac{\Omega_{c\alpha}^2}{\omega^2} \left(1 - W\left[\frac{\omega}{|k_\parallel| v_{th}}\right] \right) \right)}$$

~~$\Rightarrow$ l'approximation basse fréquence est en compétition avec l'approximation petit rayon de Larmor ($\beta_\alpha \frac{\Omega_{c\alpha}^2}{\omega^2} \gg \beta_\alpha \dots$). Cela correspond à un phénomène de démagnétisation de la particule par rapport au mouvement en y ($k_\perp \to x$). En effet $v_y \propto \frac{1}{\omega} \Leftrightarrow \dot{v}_y \propto E_y$. Par ailleurs, $n=0$, correspond à un terme de résonance Landau. Cette accélération en y est entretenue par le drift $E_y \times B_0$ qui est en x, ce qui peut rendre le processus de dérive (de démagnétisation) plus efficace que la dérive de polarisation même à β petit !?...~~

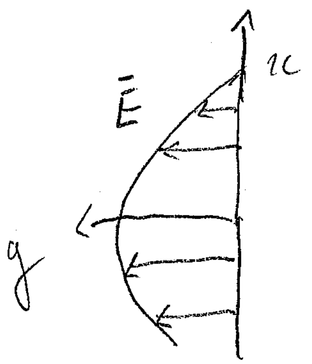
- $\omega \ll \Omega_c$, $|k_\parallel| v_{th} \ll \Omega_c$ et β quelconque :

$$\varepsilon_{yy} = \varepsilon_{xx} + 2\beta \Lambda_0'(\beta)\left(1 - W\left[\frac{\omega}{|k_\parallel| v_{th}}\right]\right)\frac{\omega_p^2}{\omega^2} - \frac{\omega_p^2}{\omega^2}(-2\beta)\sum_{n=1}^{+\infty} \Lambda_n'(\beta)\left(\frac{\omega}{\omega - n\Omega_c} + \frac{\omega}{\omega + n\Omega_c}\right)$$

$$\boxed{\varepsilon_{yy}^\alpha = \varepsilon_{xx}^\alpha + \frac{\omega_{p\alpha}^2}{\Omega_{c\alpha}^2}\left(-4\beta_\alpha \sum_{n=1}^{+\infty} \frac{\Lambda_n'(\beta)}{n^2} + 2\beta_\alpha \frac{\Omega_{c\alpha}^2}{\omega^2}\Lambda_0'(\beta_\alpha) \times \left(1 - W\left[\frac{\omega}{|k_\parallel| v_{th}}\right]\right)\right)}$$

- ~~Reprise et correction du raisonnement de Chen p 32~~

$$\left\{\begin{aligned} \bar{E} &= E_0 \cos kx\, \hat{y} \\ \dot{v}_y &= -\Omega_c v_x + \frac{q}{m} E_y(x) \\ \dot{v}_x &= +\Omega_c v_y \end{aligned}\right.$$



3) $\varepsilon_{xy}^{\alpha}$

$n = -1, 1$ termes d'ordre 0 et 1
$n = -2, 2$ termes d'ordre 1

$$\varepsilon_{xy} = -i\left[A_1\left(\tfrac{1}{2}-\beta\right)\left(1-W_I(Z_1)\right) - A_{-1}\left(\tfrac{1}{2}-\beta\right)\left(1-W_I(Z_{-1})\right)\right]$$
$$-i\left(\frac{\beta}{4}\right)\left[A_2\left(1-W_I(Z_2)\right)-A_{-2}\left(1-W_I(Z_{-2})\right)\right]\times 2$$

$$\boxed{\begin{aligned}\varepsilon_{xy} = & -i\frac{\omega_p^2}{\omega^2}\left(\frac{1}{2}-\beta\right)\frac{2\omega\Omega_c}{\omega^2-\Omega_c^2} - i\frac{\omega_p^2}{\omega^2}\frac{\beta}{4}\frac{4\omega\Omega_c}{\omega^2-4\Omega_c^2}\times 2\\ & + i\frac{\omega_p^2}{\omega}\left[\left(\frac{1}{2}-\beta\right)\left(\frac{W[Z_1]}{\omega-\Omega_c}-\frac{W[Z_{-1}]}{\omega+\Omega_c}\right)\right.\\ & \left. + \frac{\beta}{2}\left(\frac{W[Z_2]}{\omega-2\Omega_c}-\frac{W[Z_{-2}]}{\omega+2\Omega_c}\right)\right]\end{aligned}}$$

- $\omega \ll \Omega_c$ et $|k_\parallel| v_{th} \ll \Omega_c$

$$\boxed{\varepsilon_{xy}^{\alpha} = +i\frac{\omega_p^2}{\omega\Omega_c}\left(1-\frac{3\beta_\alpha}{2}\right)}$$

- idem mais β quelconque :

$$\varepsilon_{xy} = -i\frac{\omega_p^2}{\omega^2}\sum_{n=1}^{+\infty}\Lambda_n'(\beta)\, n\left(\frac{\omega}{\omega-n\Omega_c}-\frac{\omega}{\omega+n\Omega_c}\right)$$

$$= +2i\frac{\omega_p^2}{\omega\Omega_c}\sum_{n=1}^{+\infty}\Lambda_n'(\beta)$$

$$\sum_{-\infty}^{+\infty}\Lambda_n'(\beta) = 0 \quad \Rightarrow \quad \boxed{\varepsilon_{xy}^{\alpha} = -i\frac{\omega_{p\alpha}^2}{\omega\Omega_{c\alpha}}\Lambda_0'(\beta_\alpha)}$$

4) $\varepsilon^{\alpha}_{xz}$ $\quad n=-1,1$ termes d'ordre $\frac{1}{2}, \frac{3}{2}$
$\quad n=-2,2$ termes d'ordre $\frac{3}{2}$

$$\varepsilon_{xz} = -\frac{\omega_{p\alpha}^2}{\omega^2} z_0 \, sg(k_\parallel) \sum_{-\infty}^{+\infty} \frac{n}{\sqrt{\beta_\alpha}} \Lambda_n(\beta)\left(\cancel{1} - W_I(z_n)\right)$$

$$= + sg(k_\parallel) \frac{\omega_p^2}{\omega} \frac{1}{|k_\parallel| v_{th}} \left(k_\perp \frac{\cancel{v_{th}}}{\Omega_c}\right)^{-1} \sum_{-\infty}^{+\infty} n \Lambda_n(\beta) W_I(z_n)$$

$$= \frac{\Omega_c}{k_\perp k_\parallel} \frac{\omega_p^2}{v_{th}^2 \omega} \sum_{-\infty}^{+\infty} n \Lambda_n(\beta) W_I(z_n)$$

• $\omega \ll \Omega_c$ et $|k_\parallel| v_{th} \ll \Omega_c \Rightarrow W_I(z_n) \simeq -\frac{1}{z_n^2}, n \neq 0$

$$\Rightarrow \varepsilon_{xz} \simeq \frac{\Omega_c}{k_\perp k_\parallel} \frac{\omega_p^2}{v_{th}^2 \omega} \sum_{n=1}^{+\infty} -\Lambda_n(\beta) n \frac{(\omega + \cancel{n\Omega_c} + \omega - \cancel{n\Omega_c})(\cancel{\omega} + n\Omega_c - \cancel{\omega} + n\Omega_c)}{(\omega - n\Omega_c)^2 (\omega + n\Omega_c)^2} \times k_\parallel^2 v_{th}^2$$

$$= \frac{\Omega_c}{\cancel{k_\perp k_\parallel}} \frac{\omega_p^2}{\cancel{v_{th}^2 \omega}} \left(-k_\parallel^{\cancel{2}} \cancel{v_{th}^2}\right) \sum_{n=1} \Lambda_n(\beta) \frac{n \, 2\omega \times 2 n \Omega_c}{(\cancel{\omega^2} - n^2 \Omega_c^2)^2}$$

$$\boxed{\varepsilon^{\alpha}_{xz} = + \frac{\omega_{p\alpha}^2}{\Omega_c^2} \frac{k_\parallel}{k_\perp} \left(-4 \sum_{n=1}^{+\infty} \frac{\Lambda_n(\beta_\alpha)}{n^2}\right)}$$

β_α petit $\Rightarrow$ $\boxed{\varepsilon^{\alpha}_{xz} = \frac{\omega_{p\alpha}^2}{\Omega_{c\alpha}^2} \frac{k_\parallel}{k_\perp} (-2\beta_\alpha)}$

5) $\varepsilon^{\alpha}_{yz}$

$$\varepsilon_{yz} = +i\frac{\omega_p^2}{\omega^2}\, sg(k_\parallel) Z_0 \sqrt{\beta} \sum_{n=-\infty}^{+\infty} \Lambda'_n(\beta)\left(\cancel{1} - W_I(Z_n)\right)$$

$$= -i\frac{\omega_p^2}{\omega}\frac{k_\perp \cancel{v_{th}}}{k_\parallel \cancel{v_{th}}}\frac{1}{\Omega_c}\sum_{-\infty}^{+\infty}\Lambda'_n(\beta) W_I[Z_n]$$

$$= -i\frac{k_\perp}{k_\parallel}\frac{\omega_p^2}{\omega\Omega_c}\sum_{-\infty}^{+\infty}\Lambda'_n(\beta) W_I[Z_n]$$

. idem $W_I[Z_n] \simeq -\frac{1}{Z_n^2}$, $n \neq 0$

$$\Rightarrow \varepsilon_{yz} \simeq -i\frac{k_\perp}{k_\parallel}\frac{\omega_p^2}{\omega\Omega_c}\left(\Lambda'_0(\beta) W_I[Z_0] - \sum_{n=1}^{+\infty}\Lambda'_n(\beta)\frac{\overbrace{(2\omega^2 + 2n^2\Omega_c^2) k_\parallel^2 v_{th}^2}^{\sim \frac{2k_\parallel^2 v_{th}^2}{n^2\Omega_c^2}}}{(\omega^2 - n^2\Omega_c^2)^2}\right)$$

$$\boxed{\varepsilon^{\alpha}_{yz} = -i\frac{\omega_{p\alpha}^2}{\omega\Omega_{c\alpha}}\left[\frac{k_\perp}{k_\parallel}\Lambda'_0(\beta_\alpha) W_I\left[\frac{\omega}{|k_\parallel| v_{th\alpha}}\right] - \frac{k_\parallel}{k_\perp} 2\beta_\alpha \sum_{n=1}^{+\infty}\frac{\Lambda'_n(\beta_\alpha)}{n^2}\right]}$$

β_α petit $\Rightarrow$ $$\boxed{\varepsilon^{\alpha}_{yz} = +i\frac{\omega_{p\alpha}^2}{\omega\Omega_{c\alpha}}\left[\frac{k_\perp}{k_\parallel}\left(1 - \frac{3\beta_\alpha}{2}\right) W_I\left[\frac{\omega}{|k_\parallel| v_{th\alpha}}\right] + \frac{k_\parallel}{k_\perp}\beta_\alpha\right]}$$

6) $\varepsilon_{zz}^{\alpha}$

$$\varepsilon_{zz} = (1) - \frac{\omega_{p\alpha}^2}{\omega^2}\left[\sum_{-\infty}^{+\infty} z_0 z_n \Lambda_n(\beta)\left(1 - W_I(z_n)\right) - z_0^2\right]$$

$$= (1) - \frac{\omega_p^2}{\cancel{\omega^2}}\frac{\cancel{\omega^2}}{k_\parallel^2 v_{th}^2}\left[\sum_{-\infty}^{+\infty}\left(1 - n\frac{\Omega_c}{\omega}\right)\Lambda_n(\beta)\left(1 - W[z_n]\right) - 1\right]$$

$$= (1) - \frac{\omega_p^2}{k_\parallel^2 v_{th}^2}\left[\sum_{-\infty}^{+\infty}\left[\Lambda_n(\beta)\underbrace{\left(\cancel{1} - n\frac{\Omega_c}{\omega}\right)}_{\to 0} - \cancel{1} - \left(1 - n\frac{\Omega_c}{\omega}\right)\Lambda_n(\beta) W[z_n]\right]\right]$$

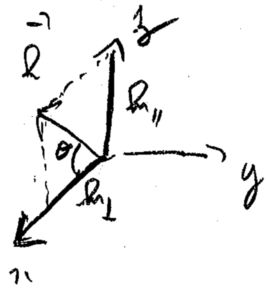
$$= (1) + \frac{\omega_p^2}{k_\parallel^2 v_{th}^2}\sum_{n=-\infty}^{+\infty}\left(1 - n\frac{\Omega_c}{\omega}\right)\Lambda_n(\beta) W_I[z_n]$$

• $\omega \ll \Omega_c$ et $|k_\parallel| v_{th} \ll |\Omega_c|$ $\Rightarrow$ $W_I[z_{n\neq 0}] \simeq \frac{-1}{z_n^2}$

$$\Rightarrow \varepsilon_{zz} \simeq (1) + \frac{\omega_p^2}{k_\parallel^2 v_{th}^2}\left[\Lambda_0(\beta) W_I[z_0] - \sum_{n=1}^{+\infty}\Lambda_n(\beta)\frac{2 k_\parallel^2 v_{th}^2}{n^2 \Omega_c^2} + \frac{\cancel{\Omega_c}}{\cancel{\omega}}\sum_{n=1}^{+\infty}\Lambda_n(\beta)\frac{4\,\cancel{\omega}\,\cancel{\Omega_c}}{n^2\Omega_c^{\cancel{4}\,2}}k_\parallel^2 v_{th}^2\right]$$

$$\boxed{\varepsilon_{zz}^{\alpha} = (1) + \frac{\omega_{p\alpha}^2}{\Omega_{c\alpha}^2}\left[\frac{\Omega_{c\alpha}^2}{k_\parallel^2 v_{th}^2}\Lambda_0(\beta_\alpha) W_I\left[\frac{\omega}{|k_\parallel| v_{th\alpha}}\right] - 2\sum_{n=1}^{+\infty}\frac{\Lambda_n(\beta_\alpha)}{n^2}\right]}$$

Ondes d'Alfvén en régime inertiel et cinétique



- $$I_T = I - \frac{\overline{\overline{kk}}}{k^2} = \begin{pmatrix} \sin\theta^2 & 0 & -\sin\theta\cos\theta \\ 0 & 1 & 0 \\ -\sin\theta \times \cos\theta & 0 & \cos^2\theta \end{pmatrix}$$

- $$\mathrm{Det}\left(\overline{\overline{\varepsilon}}(k,\omega) - \left(\frac{ck}{\omega}\right)^2 I_T\right) = \begin{vmatrix} \varepsilon_{xx} - N_{\parallel}^2 & +\varepsilon_{xy} & \varepsilon_{xz} + N_{\parallel}N_{\perp} \\ -\varepsilon_{xy} & \varepsilon_{yy} - N^2 & +\varepsilon_{yz} \\ \varepsilon_{xz} + N_{\parallel}N_{\perp} & -\varepsilon_{yz} & \varepsilon_{zz} - N_{\perp}^2 \end{vmatrix} = \Delta$$

$$\Delta = 0 \iff \boxed{\begin{aligned}&(\varepsilon_{yy} - N^2)\left[(\varepsilon_{xx} - N_{\parallel}^2)(\varepsilon_{zz} - N_{\perp}^2) - (\varepsilon_{xz} + N_{\parallel}N_{\perp})^2\right] \\ &+ \varepsilon_{yz}^2(\varepsilon_{xx} - N_{\parallel}^2) + \varepsilon_{xy}^2(\varepsilon_{zz} - N_{\perp}^2) \\ &+ 2\,\varepsilon_{xy}\varepsilon_{yz}(\varepsilon_{xz} + N_{\parallel}N_{\perp}) = 0\end{aligned}}$$

- $$\overline{\overline{\varepsilon}} = I + \frac{i}{\varepsilon_0 \omega}\overline{\overline{\sigma}} \qquad \left(e^{i(\bar{k}\cdot\bar{r} - \omega t)}\right)$$

- $$\beta_\sigma = k_\perp^2 \left(\frac{T_\sigma}{M_\sigma}\right) \Big/ \Omega_\sigma^2 = k_\perp^2 \rho_\sigma^2 \Big/ 2 \quad , \quad \Omega_\sigma = \frac{+ q_\sigma B_0}{M_\sigma}$$

$\Omega_{O^+} = 25\,Hz$ $\quad T_{O^+} = 1\,eV \rightarrow \rho_{O^+} = 23\,m$ $\quad \left(v = \sqrt{\frac{2T}{M}}\right)$

$\Omega_{H^+} = 400\,Hz$ $\quad T_{H^+} = 1\,eV \rightarrow \rho_{H^+} = 6\,m$

$\Omega_e = 700\,10^3\,Hz$ $\quad T_{e^-} = 1\,eV \rightarrow \rho_{e^-} = 0{,}13\,m$

$B_0 = 25\,000\,10^{-9}\,T$

1) $\varepsilon_{xx} = 1 + \sum_\alpha \frac{c^2}{v_{A\alpha}^2}\left(1 - \frac{3}{4}\beta_\alpha\right)$ avec $v_{A\alpha}^2 = \frac{B_0^2}{\mu_0 n_\alpha M_\alpha}$

2) $\varepsilon_{yy} = 1 + \sum_\alpha \frac{c^2}{v_{A\alpha}^2}\left(1 - \frac{11}{4}\beta_\alpha - 2\beta_\alpha \frac{\Omega_{c\alpha}^2}{\omega^2}\left(1 - W\left[\frac{\omega}{|k_\parallel| v_{th\alpha}}\right]\right)\right)$

3) $\varepsilon_{zz} = 1 + \sum_\alpha \frac{c^2}{v_{A\alpha}^2}\left(\frac{\Omega_{c\alpha}^2}{k_\parallel^2 v_{th}^2}(1-\beta_\alpha)\, W\left[\frac{\omega}{|k_\parallel| v_{th\alpha}}\right] - \beta_\alpha\right)$

4) $\varepsilon_{xy} = +i\sum_\alpha \frac{c^2}{v_{A\alpha}^2}\frac{\Omega_{c\alpha}}{\omega}\left(1-\frac{3\beta_\alpha}{2}\right)$

5) $\varepsilon_{xz} = \frac{k_\parallel}{k_\perp}\sum_\alpha \frac{c^2}{v_{A\alpha}^2}\left(-2\beta_\alpha\right)$

6) $\varepsilon_{yz} = +i\sum_\alpha \frac{c^2}{v_{A\alpha}^2}\frac{\Omega_{c\alpha}}{\omega}\left[\frac{k_\perp}{k_\parallel}\left(1-\frac{3\beta_\alpha}{2}\right)W\left[\frac{\omega}{|k_\parallel| v_{th}}\right] + \frac{k_\parallel}{k_\perp}\beta_\alpha\right]$

- $\sum_\alpha \frac{1}{v_{A\alpha}^2} = \sum_\alpha \frac{\mu_0}{B_0^2} n_\alpha M_\alpha = \frac{\mu_0 \rho}{B_0^2} = \frac{1}{v_A^2}$ (les ions sont prédominant)

- $\sum_\alpha \frac{c^2}{v_{A\alpha}^2}\beta_\alpha = \sum_\alpha \frac{c^2 k_\perp^2}{\omega^2}\frac{\omega^2}{\Omega_{c\alpha}^2}\frac{v_{th\alpha}^2}{v_{A\alpha}^2} = N_\perp^2 \sum_\alpha \frac{\omega^2}{\Omega_{c\alpha}^2}\frac{n_\alpha T_\alpha}{B_0^2/\mu_0}$

- $\sum_\alpha \frac{c^2}{v_{A\alpha}^2}\beta_\alpha\frac{\Omega_{c\alpha}^2}{\omega^2} = N_\perp^2 \sum_\alpha \frac{n_\alpha T_\alpha}{B_0^2/\mu_0} \longleftarrow \left(v_{th\alpha}^2 = \frac{T_\alpha}{M_\alpha}\right)$

- $\sum_\alpha \frac{c^2}{v_{A\alpha}^2}\Omega_{c\alpha}^2 = \sum_\alpha \frac{c^2 q^2 \cancel{B_0^2}\mu_0 n_\alpha \cancel{M_\alpha}}{M_\alpha^{\cancel{2}}\cancel{B_0^2}} = \sum_\alpha \frac{q^2 n_\alpha}{\varepsilon_0 M_\alpha} = \sum_\alpha \omega_{p\alpha}^2$

- $\sum_\alpha \frac{c^2}{v_{A\alpha}^2}\Omega_{c\alpha} = \sum_\alpha \frac{c^2 n_\alpha \cancel{M_\alpha}\mu_0}{B_0^{\cancel{2}}}\frac{q_\alpha \cancel{B_0}}{\cancel{M_\alpha}} = 0$