- Fourier spatiale et Laplace temporelle dans Vlasov :

$$-i\omega\hat{f}_{1k} - f_{1k}(0) + i\vec{k}\cdot\vec{v}\,\hat{f}_{1k} + \frac{q}{M}(\vec{v}\times\vec{B}_0)\cdot\frac{\partial}{\partial\vec{v}}\hat{f}_{1k} = -\frac{q}{M}\left(\hat{\vec{E}}_{1k}+\vec{v}\times\hat{\vec{B}}_{1k}\right)\cdot\frac{\partial}{\partial\vec{v}}f_0$$

- idem dans la loi de Faraday : $+\cancel{i}\omega\hat{\vec{B}}_{1k} + \vec{B}_{1k}(0) = \cancel{i}\vec{k}\times\vec{E}_{1k}$ (1)

$$\Rightarrow \quad \vec{v}\times\hat{\vec{B}}_{1k} = \frac{1}{\omega}\left[(\vec{v}\cdot\hat{\vec{E}}_{1k})\vec{k} - (\vec{v}\cdot\vec{k})\hat{\vec{E}}_{1k}\right] - \frac{1}{\omega}\vec{v}\times\vec{B}_{1k}(0)$$

- $\vec{k} = k_\perp\vec{x} + k_\parallel\vec{z} \quad \Rightarrow \quad \vec{k}\cdot\vec{v} = k_\perp v_\perp\cos\varphi + k_\parallel v_\parallel$; $\Omega_c = \frac{qB_0}{M}$

$$\Rightarrow \boxed{\begin{aligned} -i(\omega - k_\parallel v_\parallel - k_\perp v_\perp\cos\varphi)\hat{f}_{1k} - \Omega_c\frac{\partial}{\partial\varphi}\hat{f}_{1k} &= -\frac{q}{M}\frac{\partial f_0}{\partial\vec{v}}\cdot\underbrace{\left[\left(1-\frac{\vec{k}\cdot\vec{v}}{\omega}\right)I + \frac{\vec{k}\vec{v}}{\omega}\right]}_{L_1}\hat{\vec{E}}_{1k} \\ &+ f_{1k}(0) + \frac{q}{M\omega}\frac{\partial f_0}{\partial\vec{v}}\cdot\left(\vec{v}\times\vec{B}_{1k}(0)\right) \qquad (2)\end{aligned}}$$

- on pose $\hat{f}_{1k} = g(\vec{k},\omega,\vec{v})\; e^{-i(\omega-k_\parallel v_\parallel)\frac{\varphi}{\Omega_c}}\; e^{+ik_\perp v_\perp\sin\varphi/\Omega_c}$

$\Rightarrow (1) \Leftrightarrow \dfrac{\partial g}{\partial\varphi} = e^{+i[(\omega-k_\parallel v_\parallel)\varphi - k_\perp v_\perp\sin\varphi]/\Omega_c}$ × partie droite de (2) × $\underline{\underline{-\frac{1}{\Omega_c}}}$

- $\dfrac{\partial f_0}{\partial\vec{v}} = \dfrac{\partial f_0}{\partial v_\perp}(\cos\varphi\,\vec{x} + \sin\varphi\,\vec{y}) + \dfrac{\partial f_0}{\partial v_\parallel}\vec{z}$; "$\beta_\perp$" $= \dfrac{k_\perp v_\perp}{\omega}$; "$\beta_\parallel$" $= \dfrac{k_\parallel v_\parallel}{\omega}$

$$\left(1-\frac{\vec{k}\cdot\vec{v}}{\omega}\right)I = (1-\beta_\parallel-\beta_\perp\cos\varphi)I$$

$$\text{et}\quad \frac{\vec{k}\vec{v}}{\omega} = \begin{pmatrix}k_\perp\\0\\k_\parallel\end{pmatrix}\begin{pmatrix}v_\perp\cos\varphi\\v_\perp\sin\varphi\\v_\parallel\end{pmatrix}\frac{1}{\omega} = \begin{pmatrix}\beta_\perp\cos\varphi\,; & \beta_\perp\sin\varphi\,; & \frac{k_\perp}{k_\parallel}\beta_\parallel\\ 0\,; & 0\,; & 0\\ \frac{k_\parallel}{k_\perp}\beta_\perp\cos\varphi\,; & \frac{k_\parallel}{k_\perp}\beta_\perp\sin\varphi\,; & \beta_\parallel\end{pmatrix}$$

$$\Rightarrow L_1 = \begin{pmatrix} \frac{\partial f_0}{\partial v_\perp}\cos\varphi \\ \frac{\partial f_0}{\partial v_\perp}\sin\varphi \\ \frac{\partial f_0}{\partial v_\parallel} \end{pmatrix} \cdot \begin{pmatrix} 1-\beta_\parallel \;; & \beta_\perp \sin\varphi \;; & \frac{k_\perp}{k_\parallel}\beta_\parallel \\ 0 \;; & 1-\beta_\parallel-\beta_\perp\cos\varphi \;; & 0 \\ \frac{k_\parallel}{k_\perp}\beta_\perp\cos\varphi \;; & \frac{k_\parallel}{k_\perp}\beta_\perp\sin\varphi \;; & 1-\beta_\perp\cos\varphi \end{pmatrix}$$

$$\Rightarrow \boxed{\begin{aligned} L_1(\vec{x}) &= \left(\frac{\partial f_0}{\partial v_\perp}(1-\beta_\parallel) + \frac{\partial f_0}{\partial v_\parallel}\frac{k_\parallel}{k_\perp}\beta_\perp\right)_{\alpha_x}\cos\varphi \\ L_1(\vec{y}) &= \left(\frac{\partial f_0}{\partial v_\perp}(1-\beta_\parallel) + \frac{\partial f_0}{\partial v_\parallel}\frac{k_\parallel}{k_\perp}\beta_\perp\right)_{\alpha_y}\sin\varphi \\ L_1(\vec{z}) &= \frac{\partial f_0}{\partial v_\parallel} + \left(\frac{\partial f_0}{\partial v_\perp}\frac{k_\perp}{k_\parallel}\beta_\parallel - \frac{\partial f_0}{\partial v_\parallel}\beta_\perp\right)_{\alpha_z}\cos\varphi \end{aligned}}$$

/!\ "$\beta_\perp$" et "$\beta_\parallel$" définis avec ω et non Ω_c.

- $e^{-i\frac{k_\perp v_\perp}{\Omega_c}\sin\varphi} = e^{-i\beta\sin\varphi} = \sum_{n=-\infty}^{+\infty} J_n(\beta)\, e^{-in\varphi}$

$$\Rightarrow \frac{\partial g}{\partial\varphi} = \sum_{n=-\infty}^{+\infty} J_n(\beta)\, e^{i(\omega - k_\parallel v_\parallel - n\Omega_c)\frac{\varphi}{\Omega_c}} \times$$

$$\left[-\frac{q}{M} L_1(\hat{\vec{E}}_{1k}) + \underbrace{f_{1k}(0) + \frac{q}{M\omega}\frac{\partial f_0}{\partial \vec{v}}\cdot(\vec{v}\times\vec{B}_{1k}(0))}_{C_i}\right] \times \left(\frac{-1}{\Omega_c}\right)$$

③

$$\Rightarrow g(\varphi) = g(\varphi=0) + \int_0^{\varphi} d\varphi' \sum_{-\infty}^{+\infty} J_n(\beta)\, e^{i(\omega - k_\parallel v_\parallel - n\Omega_c)\frac{\varphi'}{\Omega_c}} \left[-\frac{q}{M} L_1 \cdot \hat{\vec{E}}_{1k} + C_i\right] \times \frac{-1}{\Omega_c}$$

$$\text{or} \quad g(0) = \hat{f}_{1k}(0) = \hat{f}_{1k}(2\pi) = g(2\pi)\, e^{-i(\omega - k_\parallel v_\parallel)\frac{2\pi}{\Omega_c}}$$

$$\Rightarrow g(0) \Big[\underbrace{e^{+i(\omega - k_\parallel v_\parallel)\frac{2\pi}{\Omega_c}} - 1}_{d}\Big] = \int_0^{2\pi} d\varphi' \sum_{-\infty}^{+\infty} J_n\, e^{i(\omega - k_\parallel v_\parallel - n\Omega_c)\frac{\varphi'}{\Omega_c}} \left[-\frac{q}{M} L_1 \cdot \hat{\vec{E}}_{1k} + C_i\right] \times \frac{-1}{\Omega_c}$$

- $$I_n^{\varepsilon}(\varphi) = \int_0^{\varphi} e^{i(\omega - k_\parallel - n\Omega_c)\frac{\varphi'}{\Omega_c}} e^{i\varepsilon\varphi'} = \frac{e^{i(\omega - k_\parallel^{v_\parallel})\frac{\varphi}{\Omega_c} - i(n-\varepsilon)\varphi} - 1}{i(\omega - k_\parallel)/\Omega_c - i(n-\varepsilon)}$$

- $$\left.\begin{matrix} K_n(\varphi) \\ L_n(\varphi) \end{matrix}\right\} = \int_0^{\varphi} d\varphi'\, e^{i(\omega - k_\parallel v_\parallel - n\Omega_c)\frac{\varphi'}{\Omega_c}} \begin{cases} \cos\varphi \\ \sin\varphi \end{cases}$$

$$= \left(\frac{e^{i(\omega - k_\parallel^{v_\parallel})\frac{\varphi}{\Omega_c} - i(n-1)\varphi} - 1}{i(\omega - k_\parallel)/\Omega_c - i(n-1)} \pm \frac{e^{i(\omega - k_\parallel^{v_\parallel})\frac{\varphi}{\Omega_c} - i(n+1)\varphi} - 1}{i(\omega - k_\parallel)/\Omega_c - i(n+1)} \right) \times \begin{cases} 1/2 \\ -i/2 \end{cases}$$

$$\Rightarrow g(0) = \left\{ -\frac{q}{M} \sum_{n=-\infty}^{+\infty} J_n \left[\alpha_x K_n(2\pi),\ \alpha_y L_n(2\pi),\ \alpha_z K_n(2\pi) + \text{err}_1\right] \cdot \hat{\vec{E}}_{1k} \Big/ \underbrace{[\ldots]}_{= d} \right.$$

$$\frac{D_i(2\pi)}{d} \left\{ + \int_0^{2\pi} d\varphi' \sum_{-\infty}^{+\infty} J_n(\beta)\, e^{i(\omega - k_\parallel v_\parallel - n\Omega_c)\frac{\varphi'}{\Omega_c}}\, C_i(\vec{v}; k, \omega) \Big/ \left[e^{i(\omega - k_\parallel v_\parallel)\frac{2\pi}{\Omega_c}} - 1\right] \right\} d \Big\} \times \left(-\frac{1}{\Omega_c}\right)$$

- $$\left.\begin{matrix} K_n(\varphi) + K_n(2\pi)/d \\ L_n(\varphi) + L_n(2\pi)/d \end{matrix}\right\} = \left(\frac{e^{i(\omega - k_\parallel)\frac{\varphi}{\Omega_c} - i(n-1)\varphi}}{i(\omega - k_\parallel^{v_\parallel})/\Omega_c - i(n-1)} \pm \frac{e^{i(\omega - k_\parallel v_\parallel)\frac{\varphi}{\Omega_c} - i(n+1)\varphi}}{i(\omega - k_\parallel^{v_\parallel})/\Omega_c - i(n+1)} \right) \times \begin{cases} 1/2 \\ -i/2 \end{cases}$$

$$= \left(F_{n-1} \pm F_{n+1}\right) \times \begin{cases} 1/2 \\ -i/2 \end{cases}$$

$$\Rightarrow g(\varphi) = -\frac{q}{M}\sum_{n=-\infty}^{+\infty} J_n \left[\alpha_x (F_{n-1}+F_{n+1})/2 \,,\ \alpha_y (F_{n-1}-F_{n+1})/2i \,,\ \begin{vmatrix} \alpha_z (F_{n-1}+F_{n+1})/2 \\ + \mathrm{err}_2 \end{vmatrix} \right] \cdot \hat{\vec{E}}_{1k} \times \left(-\frac{1}{\Omega_c}\right) +$$

$$\left(D_i(\varphi) + D_i(2\pi)/d \right)\left(\frac{-1}{\Omega_c}\right)$$

$$= \left(-\frac{q}{M}\sum_{n=-\infty}^{+\infty} F_n \left[\alpha_x \frac{n}{\beta} J_n \,,\ \begin{vmatrix} \alpha_y J_n' \\ \times i \end{vmatrix} ,\ \begin{vmatrix} + \frac{\partial f_0}{\partial v_\parallel} J_n \\ \alpha_z \frac{n}{\beta} J_n \end{vmatrix} \right] \cdot \hat{\vec{E}}_{1k} + D_i(\varphi) + D_i(2\pi)/d \right) \times \frac{-1}{\Omega_c}$$

$$\boxed{\begin{aligned} \Rightarrow \hat{f}_{1k} &= -\frac{q}{M}\sum_{n=-\infty}^{+\infty} e^{-i(\omega - k_\parallel v_\parallel)\frac{\varphi}{\Omega_c}} F_n e^{+i\beta \sin\varphi} \left[\alpha_x \frac{n}{\beta} J_n, \alpha_y J_n' \times i, \begin{vmatrix} \alpha_z \frac{n}{\beta} J_n \\ + \frac{\partial f_0}{\partial v_\parallel} J_n \end{vmatrix} \right] \cdot \hat{\vec{E}}_{1k} \\ &\quad + e^{-i(\omega - k_\parallel v_\parallel)\frac{\varphi}{\Omega_c}} e^{+i\beta\sin\varphi} \left(D_i(\varphi) + D_i(2\pi)/d \right) \times \left(-\frac{1}{\Omega_c}\right) \\ &= L_2 \cdot \hat{\vec{E}}_{1k} + b_i \end{aligned}}$$

$\Rightarrow$ /!\

$$\text{et } \hat{\vec{J}}_{1k} = \sum_s \underbrace{\int d\vec{v}\, q\vec{v}\, L_2}_{\sigma_s} \cdot \hat{\vec{E}}_{1k} + \sum \underbrace{\int d\vec{v}\, q\vec{v}\, b_i}_{\vec{j}_{ks}} \qquad (3)$$

- Loi d'Ampère : $i\vec{k} \times \hat{\vec{B}}_{1k} = \mu_0 \hat{\vec{J}}_{1k} + \frac{1}{c^2}\left[(-i\omega)\hat{\vec{E}}_{1k} - E_{1k}(0)\right]$ (4), à combiner avec (1) : $i\vec{k} \times (\vec{k} \times \hat{\vec{E}}_{1k} - \vec{B}_{1k}(0))\frac{c^2}{\omega} = \frac{1}{\varepsilon_0}\sum_s(\sigma_s \hat{\vec{E}}_{1k} + \vec{j}_{ks}) - i\omega\hat{\vec{E}}_{1k} - \hat{E}_{1k}(0)$

$$\boxed{\begin{aligned} \underbrace{\left[\frac{c^2}{\omega^2}(\vec{k}\times\vec{k}\times\cdot) + \underbrace{\frac{i}{\varepsilon_0\omega}\sum_s \sigma_s + I}_{\overline{\overline{\varepsilon}}} \right]}_{\overline{\overline{D}}} \cdot \hat{\vec{E}}_{1k} &= -\frac{i}{\varepsilon_0\omega}\sum \hat{\vec{j}}_{ks} \\ &\quad + \frac{c^2}{\omega^2}\vec{k}\times\vec{B}_{1k}(0) \\ &\quad - \frac{1}{i\omega}\vec{E}_{1k}(0) \end{aligned}}$$

$$\bullet \quad err_1 = \frac{\partial f_0}{\partial v_\parallel} \times \frac{\alpha}{i(\omega - k_\parallel v_\parallel)/\Omega_c - i n}$$

$$\bullet \quad err_2 = \cancel{err_1/\alpha} + \frac{e^{i(\omega - k_\parallel v_\parallel)\frac{\varphi}{\Omega_c} - i n \varphi} \cancel{- 1}}{i(\omega - k_\parallel v_\parallel)/\Omega_c - i n} \times \frac{\partial f_0}{\partial v_\parallel} = \frac{\partial f_0}{\partial v_\parallel} F_n$$

$$\bullet \quad \varepsilon_{xx} = \int d\vec{v} \sum_{n=-\infty}^{+\infty} \frac{n}{\beta} J_n(\beta) \frac{e^{i\beta \sin\varphi}\, e^{-in\varphi}\, \cancel{\Omega_c}}{-i(n\Omega_c + k_\parallel v_\parallel - \omega)} \frac{e^{i\varphi} + e^{-i\varphi}}{2} v_\perp \times \frac{-1}{\cancel{\Omega_c}}$$

$$\times \left[\frac{\partial f_0}{\partial v_\perp}(\omega - k_\parallel v_\parallel) + \frac{\partial f_0}{\partial v_\parallel} k_\parallel v_\perp \right] \frac{1}{\omega} \times \frac{\cancel{i}}{\varepsilon_0 \omega} \times n_s q \times \left(+\frac{q}{M}\right) + 1$$

$$= -\frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{+\infty} \int d\vec{v} \frac{n}{\beta} J_n(\beta) \frac{1}{2}\left(J_{n-1}(\beta) + J_{n+1}(\beta)\right) \left[\frac{\partial f_0}{\partial v_\perp}(-1) v_\perp + \left(\frac{\partial f_0}{\partial v_\perp} n\Omega_c + \frac{\partial f_0}{\partial v_\parallel} k_\parallel v_\perp \right) \frac{v_\perp}{n\Omega_c + k_\parallel v_\parallel - \omega} \right] + 1$$

$$= -\frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{+\infty} \int d\vec{v} \left(\frac{n}{\beta} J_n(\beta) \right)^2 \left[\frac{\partial f_0}{\partial v_\perp}(-1) v_\perp + \frac{v_\perp^2}{n\Omega_c + k_\parallel v_\parallel - \omega} \times \left(\frac{n\Omega_c}{v_\perp} \frac{\partial f_0}{\partial v_\perp} + k_\parallel \frac{\partial f_0}{\partial v_\parallel} \right) \right] + 1$$

or $\sum n^2 J_n^2(\beta) = \frac{\beta^2}{2}$ et $\int d\varphi\, dv_\parallel\, dv_\perp\; v_\perp^2 \frac{\partial f_0}{\partial v_\perp} = -2$

$$\Rightarrow \boxed{\varepsilon_{xx} = 1 - \frac{\omega_{ps}^2}{\omega^2} - \frac{\omega_{ps}^2}{\omega^2} \sum_{-\infty}^{+\infty} \int d\vec{v} \left(\frac{n\Omega_c}{v_\perp} \frac{\partial f_0}{\partial v_\perp} + k_\parallel \frac{\partial f_0}{\partial v_\parallel} \right) \times \frac{n^2 \Omega_c^2}{k_\perp^2} J_n^2(\beta) \times \frac{1}{n\Omega_c + k_\parallel v_\parallel - \omega}}$$

(p 51, Ichimaru)

$$\bullet\ \varepsilon_{yx} = \int d\vec{v} \sum_{n=-\infty}^{+\infty} \frac{n}{\beta} J_n(\beta) \frac{e^{i\beta \sin\varphi}\, e^{-in\varphi}\, \cancel{\Omega_c}}{\cancel{i}(n\Omega_c + k_\parallel v_\parallel - \omega)} \; \frac{e^{i\varphi} - e^{-i\varphi}}{2\cancel{i}}\, v_\perp \times \frac{+1}{\cancel{\Omega_c}}$$

$$\times \left[\frac{\partial f_0}{\partial v_\perp}(\omega - k_\parallel v_\parallel - n\Omega_c + n\Omega_c) + \frac{\partial f_0}{\partial v_\parallel} k_\parallel v_\perp\right] \frac{1}{\omega} \times \frac{i}{\omega\, \varepsilon_0} \times n_s q \times \left(+\frac{q}{M}\right) \quad \omega_{ps}^2$$

$$= i \frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{+\infty} \int d\vec{v}\, \frac{n}{\beta} J_n(\beta) \frac{1}{2}\left[J_{n-1} - J_{n+1}\right] \left(- v_\perp \frac{\partial f_0}{\partial v_\perp} + \frac{v_\perp^2}{n\Omega_c + k_\parallel v_\parallel - \omega} \times \left[\frac{n\Omega_c}{v_\perp}\frac{\partial f_0}{\partial v_\perp} + k_\parallel \frac{\partial f_0}{\partial v_\parallel}\right]\right)$$

$$= i \frac{\omega_{ps}^2}{\omega^2} \int d\vec{v}\, \frac{n\Omega_c}{k_\perp} J_n(\beta) J_n'(\beta) \left[\underbrace{-\frac{\partial f_0}{\partial v_\perp}}_{\to 0 \text{ car } \sum n J_n J_n' = 0} + \frac{v_\perp}{n\Omega_c + k_\parallel v_\parallel - \omega} \times \left(\frac{n\Omega_c}{v_\perp}\frac{\partial f_0}{\partial v_\perp} + k_\parallel \frac{\partial f_0}{\partial v_\parallel}\right)\right]$$

$$\Rightarrow \boxed{\varepsilon_{yx} = -\frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{+\infty} \int d\vec{v} \left(\frac{n\Omega_c}{v_\perp}\frac{\partial f_0}{\partial v_\perp} + k_\parallel \frac{\partial f_0}{\partial v_\parallel}\right) \times -i v_\perp \frac{n\Omega_c}{k_\perp} J_n(\beta) J_n'(\beta) \times \frac{1}{n\Omega_c + k_\parallel v_\parallel - \omega}}$$

$$\bullet\ \varepsilon_{zx} = \int d\vec{v} \sum_{-\infty}^{+\infty} \frac{n}{\beta} J_n(\beta) \frac{e^{i\beta\sin\varphi}\, e^{-in\varphi}\, \cancel{\Omega_c}}{\cancel{i}(n\Omega_c + k_\parallel v_\parallel - \omega)}\, v_\parallel \times \frac{-1}{\cancel{\Omega_c}} \times \cancel{-i}\, \frac{\omega_{ps}^2}{\omega^2}$$

$$\times \left[\frac{\partial f_0}{\partial v_\perp}(\omega - k_\parallel v_\parallel - n\Omega_c + n\Omega_c) + \frac{\partial f_0}{\partial v_\parallel} k_\parallel v_\perp\right]$$

$$\varepsilon_{zx} = -\frac{\omega_{ps}^2}{\omega^2}\int d\vec{v} \sum_{n=-\infty}^{+\infty} \frac{n\Omega_c}{k_\perp} J_n^2(\beta)\, v_\parallel \left[\underbrace{-\frac{1}{v_\perp}\frac{\partial f_0}{\partial v_\perp}}_{0} + \frac{1}{(n\Omega_c + k_\parallel v_\parallel - \omega)} \times \left(\frac{n\Omega_c}{v_\perp}\frac{\partial f_0}{\partial v_\perp} + k_\parallel \frac{\partial f_0}{\partial v_\parallel}\right)\right]$$

$$\left(\sum n J_n^2 = 0 \right)$$

$$\Rightarrow \boxed{\begin{aligned}\varepsilon_{zx} = & -\frac{\omega_{ps}^2}{\omega^2}\sum_{n=-\infty}^{+\infty}\int d\vec{v}\left(\frac{n\Omega_c}{v_\perp}\frac{\partial f_0}{\partial v_\perp} + k_\parallel \frac{\partial f_0}{\partial v_\parallel}\right) \\ & \times v_\parallel \frac{n\Omega_c}{k_\perp} J_n^2(\beta) \times \frac{1}{n\Omega_c + k_\parallel v_\parallel - \omega}\end{aligned}} \blacksquare$$

- $$\varepsilon_{xy} = \int d\vec{v} \sum_{-\infty}^{+\infty} i J_n'(\beta) \frac{e^{i\beta\sin\varphi} e^{-in\varphi} \cancel{\Omega_c}}{\cancel{-i}(n\Omega_c + k_\parallel v_\parallel - \omega)} \frac{e^{i\varphi} + e^{-i\varphi}}{2} v_\perp \times \frac{-1}{\cancel{\Omega_c}} \times \cancel{-i} \frac{\omega_{ps}^2}{\omega^2} \times [\dots]$$

$$= -\frac{\omega_{ps}^2}{\omega^2}\sum_{n=-\infty}^{+\infty}\int d\vec{v}\, i J_n'(\beta) J_n(\beta) \frac{n\Omega_c}{k_\perp} v_\perp \left[\underbrace{-\frac{1}{v_\perp}\frac{\partial f_0}{\partial v_\perp}}_{\to 0 \text{ car } \sum n J_n' J_n = 0} + \frac{1}{(n\Omega_c + k_\parallel v_\parallel - \omega)} \times \left(\frac{n\Omega_c}{v_\perp}\frac{\partial f_0}{\partial v_\perp} + k_\parallel \frac{\partial f_0}{\partial v_\parallel}\right)\right]$$

$$\Rightarrow \boxed{\varepsilon_{xy} = -\varepsilon_{yx}} \blacksquare$$

- $$\varepsilon_{yy} = \int d\vec{v} \sum_{-\infty}^{+\infty} \cancel{i} J_n'(\beta) \frac{e^{i\beta\sin\varphi} e^{-in\varphi} \cancel{\Omega_c}}{\cancel{-i}(n\Omega_c + k_\parallel v_\parallel - \omega)} \underbrace{\frac{e^{i\varphi} - e^{-i\varphi}}{2\cancel{i}}}_{\to J_n'(\beta)} v_\perp \times \frac{-1}{\cancel{\Omega_c}} \times \cancel{-i} \frac{\omega_{ps}^2}{\omega^2} \times [\dots] + 1$$

$$= -\frac{\omega_{ps}^2}{\omega^2}\sum_{n=-\infty}^{+\infty}\int d\vec{v}\, v_\perp^2 J_n'^2(\beta) \left[-\frac{1}{v_\perp}\frac{\partial f_0}{\partial v_\perp} + \frac{1}{(n\Omega_c + k_\parallel v_\parallel - \omega)} \times \left(\frac{n\Omega_c}{v_\perp}\frac{\partial f_0}{\partial v_\perp} + k_\parallel \frac{\partial f_0}{\partial v_\parallel}\right)\right] + 1$$

or $\sum J_n'^2 = \sum \frac{1}{4}\Big(\underbrace{J_{n-1}^2}_{1} + \underbrace{J_{n+1}^2}_{1} - 2\underbrace{J_{n-1} J_{n+1}}_{0}\Big) = \frac{1}{2}$ (Théorème d'addition de Neumann)

et $\int d\varphi\, dv_\parallel\, dv_\perp\, v_\perp^2 \frac{\partial f_0}{\partial v_\perp} = -2$

$$\Rightarrow \boxed{\begin{aligned}\varepsilon_{yy} = 1 - \frac{\omega_{ps}^2}{\omega^2} - \frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{+\infty} \int d\vec{v} \left(\frac{n\Omega_c}{v_\perp} \frac{\partial f_0}{\partial v_\perp} + k_\parallel \frac{\partial f_0}{\partial v_\parallel} \right) \\ \times \left(v_\perp J_n'(\beta) \right)^2 \times \frac{1}{n\Omega_c + k_\parallel v_\parallel - \omega}\end{aligned}} \blacksquare$$

- $$\varepsilon_{zy} = \int d\vec{v} \sum_{-\infty}^{+\infty} i J_n'(\beta) \frac{e^{i\beta \sin\varphi} e^{-in\varphi} \cancel{\Omega_c}}{\cancel{-i}(n\Omega_c + k_\parallel v_\parallel - \omega)} v_\parallel \times \frac{-1}{\cancel{\Omega_c}} \times \cancel{-i} \frac{\omega_{ps}^2}{\omega^2} \times [\dots]$$

$$= -\frac{\omega_{ps}^2}{\omega^2} \sum_{-\infty}^{+\infty} \int d\vec{v}\; i J_n'(\beta) J_n(\beta) v_\parallel v_\perp \Bigg[\underbrace{-\frac{1}{v_\perp} \frac{\partial f_0}{\partial v_\perp}}_{\hookrightarrow 0 \ (\sum J_n J_n' = 0)} + \frac{\left(\frac{n\Omega_c}{v_\perp} \frac{\partial f_0}{\partial v_\perp} + k_\parallel \frac{\partial f_0}{\partial v_\parallel} \right)}{n\Omega_c + k_\parallel v_\parallel - \omega} \Bigg]$$

$$\Rightarrow \boxed{\begin{aligned}\varepsilon_{zy} = -\frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{+\infty} \int d\vec{v} \left(\frac{n\Omega_c}{v_\perp} \frac{\partial f_0}{\partial v_\perp} + k_\parallel \frac{\partial f_0}{\partial v_\parallel} \right) \\ \times\; i J_n(\beta) J_n'(\beta) \times \frac{1}{n\Omega_c + k_\parallel v_\parallel - \omega}\end{aligned}} \blacksquare$$

- $$\varepsilon_{xz} = \sum_{n=-\infty}^{+\infty} \int d\vec{v} \left(\frac{-1}{\cancel{\Omega_c}} \right) \left(\cancel{-i} \frac{\omega_{ps}^2}{\omega^2} \right) \frac{e^{i\beta \sin\varphi} e^{-in\varphi} \cancel{\Omega_c}}{\cancel{-i}(n\Omega_c + k_\parallel v_\parallel - \omega)} v_\perp \frac{e^{i\varphi} + e^{-i\varphi}}{2} \times$$

$$\times \left[\frac{n}{\beta} J_n(\beta) \left(\frac{\partial f_0}{\partial v_\perp} k_\perp v_\parallel - \frac{\partial f_0}{\partial v_\parallel} k_\perp v_\perp \right) + J_n(\beta) \frac{\partial f_0}{\partial v_\parallel} \omega \right]$$

$$= -\frac{\omega_{ps}^2}{\omega^2} \sum_{-\infty}^{+\infty} \int \Bigg[\underbrace{v_\perp \frac{n}{\beta}}_{\frac{n\Omega_c}{k_\perp}} J_n^2(\beta) \Bigg[\underbrace{-\frac{\partial f_0}{\partial v_\parallel}}_{\downarrow\, 0} + \frac{v_\parallel}{n\Omega_c + k_\parallel v_\parallel - \omega} \left(\frac{n\Omega_c}{v_\perp} \frac{\partial f_0}{\partial v_\perp} + k_\parallel \frac{\partial f_0}{\partial v_\parallel} \right) \Bigg] \Bigg] d\vec{v}$$

$$\Rightarrow \boxed{\varepsilon_{xz} = \varepsilon_{zx}} \blacksquare$$

$$\bullet\ \varepsilon_{yz} = -\frac{\omega_{ps}^2}{\omega^2}\sum_{-\infty}^{+\infty}\int d\vec{v}\ \frac{e^{i\beta\sin\varphi}e^{-in\varphi}}{n\Omega_c+k_\parallel v_\parallel-\omega}\ v_\perp\ \frac{e^{i\varphi}-e^{-i\varphi}}{2i}\times J_n(\beta)\times$$

$$\times\Big[\underbrace{-\frac{\partial f_0}{\partial v_\parallel}}_{\to 0}+\frac{v_\parallel}{n\Omega_c+k_\parallel v_\parallel-\omega}\Big(\frac{n\Omega_c}{v_\perp}\frac{\partial f_0}{\partial v_\perp}+k_\parallel\frac{\partial f_0}{\partial v_\parallel}\Big)\Big](n\Omega_c+k_\parallel v_\parallel-\omega)$$

$$= -\frac{\omega_{ps}^2}{\omega^2}\sum_{-\infty}^{+\infty}\int d\vec{v}\ v_\perp\ J_n(\beta)\,J_n'(\beta)(-i)\,v_\parallel\Big(\frac{n\Omega_c}{v_\perp}\frac{\partial f_0}{\partial v_\perp}+k_\parallel\frac{\partial f_0}{\partial v_\parallel}\Big)$$

$$\Rightarrow \boxed{\varepsilon_{yz} = -\ \varepsilon_{zy}}\ \blacksquare$$

$$\bullet\ \varepsilon_{zz} = -\frac{\omega_{ps}^2}{\omega^2}\sum_{-\infty}^{+\infty}\int d\vec{v}\ e^{i\beta\sin\varphi}e^{-in\varphi}\ v_\parallel\ \ J_n(\beta)$$

$$\times\Big[\underbrace{-\frac{\partial f_0}{\partial v_\parallel}}_{\to +1}+\frac{v_\parallel}{n\Omega_c+k_\parallel v_\parallel-\omega}\Big(\frac{n\Omega_c}{v_\perp}\frac{\partial f_0}{\partial v_\perp}+k_\parallel\frac{\partial f_0}{\partial v_\parallel}\Big)\Big]+1$$

$$\Rightarrow \boxed{\varepsilon_{zz} = 1-\frac{\omega_{ps}^2}{\omega^2}-\frac{\omega_{ps}^2}{\omega^2}\sum_{-\infty}^{+\infty}\int d\vec{v}\Big(\frac{n\Omega_c}{v_\perp}\frac{\partial f_0}{\partial v_\perp}+k_\parallel\frac{\partial f_0}{\partial v_\parallel}\Big)\times v_\parallel^2\ J_n^2(\beta)\times\frac{1}{n\Omega_c+k_\parallel v_\parallel-\omega}}\ \blacksquare$$

Expression de $\vec{j}_{k}$

- On supposera $B_{1k}(0) = E_{1k}(0) = 0$; seul $f_{1k}(0) \neq 0$.
- On choisit $f_{1k}(0) = n_{1k}(0) \times \dfrac{1}{w_\perp}\,\delta(v_\perp - w_\perp)\,\delta(v_\parallel - w_\parallel)\,\delta(\varphi - \varphi_0)$

$$\Rightarrow D_i(\varphi) = \int_0^{\varphi} d\varphi' \sum_{-\infty}^{+\infty} J_n(\beta)\, e^{i(\omega - k_\parallel v_\parallel - n\Omega_c)\frac{\varphi'}{\Omega_c}} f_{1k}(0)$$

$$= \underbrace{\frac{1}{w_\perp}\delta(v_\perp - w_\perp)\delta(v_\parallel - w_\parallel)\, n_{1k}(0)}_{d_i}\, e^{i(\omega - k_\parallel w_\parallel)\frac{\varphi_0}{\Omega_c}} \times \begin{cases} 0 & \text{si } 0 < \varphi < \varphi_0 \\ \underbrace{\sum_{-\infty}^{+\infty} J_n(\beta_\perp)\, e^{-in\varphi_0}}_{= e^{-i\beta_\perp \sin\varphi_0}} & \text{sinon} \end{cases}$$

- $D_i(2\pi)/d = d_i\, e^{i(\omega - k_\parallel w_\parallel)\frac{\varphi_0}{\Omega_c}}\, e^{-\beta_\perp \sin\varphi_0}/d$, $\beta_\perp = \dfrac{k_\perp w_\perp}{\Omega_c}$

$$\Rightarrow b_i = d_i\, e^{i(\omega - k_\parallel w_\parallel)\frac{\varphi_0}{\Omega_c}}\, e^{-i\beta_\perp \sin\varphi_0} \times \begin{cases} \dfrac{1}{d} & \text{si } 0 < \varphi < \varphi_0 \\ 1 + \dfrac{1}{d} & \text{sinon} \end{cases}$$

$$\times\; e^{-i(\omega - k_\parallel w_\parallel)\frac{\varphi}{\Omega_c}}\, e^{+i\beta_\perp \sin\varphi}$$

$$\bullet\ f_1(\varphi, v_\perp, v_\parallel) = B_0^{-1} \left\{ \sum_{n=-\infty}^{+\infty} \frac{e^{i\beta \sin\varphi}\, e^{-in\varphi}}{i\left(\frac{\omega - k_\parallel v_\parallel}{\Omega_c} - n\right)} \times \right.$$

$$\left[\frac{n}{\beta} J_n(\beta) \left(\frac{\partial f_0}{\partial v_\perp}\left(1 - \frac{k_\parallel v_\parallel}{\omega}\right) + \frac{\partial f_0}{\partial v_\parallel} \frac{k_\parallel v_\perp}{\omega} \right), \right.$$

$$i J_n'(\beta) \left(\frac{\partial f_0}{\partial v_\perp}\left(1 - \frac{k_\parallel v_\parallel}{\omega}\right) + \frac{\partial f_0}{\partial v_\parallel} \frac{k_\parallel v_\perp}{\omega} \right),$$

$$\left. \left. \frac{n}{\beta} J_n(\beta) \left(\frac{\partial f_0}{\partial v_\perp} \frac{k_\perp v_\parallel}{\omega} - \frac{\partial f_0}{\partial v_\parallel} \frac{k_\perp v_\perp}{\omega} \right) + J_n(\beta) \frac{\partial f_0}{\partial v_\parallel} \right] \right\} \cdot \overline{E_1}$$

$$= \frac{\Omega_c}{\omega B_0} \left\{ \sum_{n=-\infty}^{+\infty} \frac{e^{i\beta \sin\varphi}\, e^{-in\varphi}}{i\left(\frac{\omega - k_\parallel v_\parallel}{\Omega_c} - n\right)} \times \right.$$

$$\left[\frac{n}{\beta} J_n(\beta) \left(\frac{\partial f_0}{\partial v_\perp}\left(\frac{\omega - k_\parallel v_\parallel}{\Omega_c} - n\right) + n \frac{\partial f_0}{\partial v_\perp} + \frac{\partial f_0}{\partial v_\parallel} \frac{k_\parallel v_\perp}{\Omega_c} \right), \right.$$

$$i J_n'(\beta) \left(\rule{6cm}{0.4pt} \right),$$

$$\frac{n}{\beta} J_n(\beta) \frac{\partial f_0}{\partial v_\perp} \frac{k_\perp v_\parallel}{\Omega_c} + J_n(\beta) \frac{\partial f_0}{\partial v_\parallel} \left(\frac{\omega}{\Omega_c} - n - \frac{k_\parallel v_\parallel}{\Omega_c} \right)$$

$$\left. \left. + J_n(\beta) \frac{\partial f_0}{\partial v_\parallel} \frac{k_\parallel v_\parallel}{\Omega_c} \right] \right\} \cdot \overline{E_1}$$

$$\bullet\ \sum_{n=-\infty}^{+\infty} J_n(\beta)\, e^{-in\varphi} = e^{-i\beta \sin\varphi}$$

$$\Rightarrow \sum_{n=-\infty}^{+\infty} J_n'(\beta)\, e^{-in\varphi} = -i \sin\varphi\, e^{-i\beta \sin\varphi}$$

$$\text{et } \sum_{-\infty}^{+\infty} -in\, J_n(\beta)\, e^{-in\varphi} = -i\beta \cos\varphi\, e^{-i\beta \sin\varphi}$$

$$\bullet \quad f_1(\varphi, v_\perp, v_\parallel) = \frac{\Omega_c}{\omega B_0} \left\{ \sum_{-\infty}^{+\infty} \frac{e^{i\beta \sin\varphi}\, e^{-in\varphi}}{\left(\dfrac{\omega - k_\parallel v_\parallel}{\Omega_c} - n\right)} \times \right.$$

$$\left[-i \frac{n}{\beta} J_n(\beta) \left(n \frac{\partial f_0}{\partial v_\perp} + \frac{\partial f_0}{\partial v_\parallel} \frac{k_\parallel v_\perp}{\Omega_c} \right), \right.$$

$$J'_n(\beta) \left(n \frac{\partial f_0}{\partial v_\perp} + \frac{\partial f_0}{\partial v_\parallel} \frac{k_\parallel v_\perp}{\Omega_c} \right),$$

$$\left. \left. -i J_n(\beta) \left(n \frac{\partial f_0}{\partial v_\perp} \frac{v_\parallel}{v_\perp} + \frac{\partial f_0}{\partial v_\parallel} \frac{k_\parallel v_\parallel}{\Omega_c} \right) \right] \right\} \cdot \overline{E}_1$$

$$+$$

$$-i \frac{\Omega_c}{\omega B_0} \left[\frac{\partial f_0}{\partial v_\perp} \cos\varphi, \frac{\partial f_0}{\partial v_\perp} \sin\varphi, \frac{\partial f_0}{\partial v_\parallel} \right] \cdot \overline{E}_1$$

$$\boxed{\begin{aligned} f_1(\varphi, v_\perp, v_\parallel) = -i \frac{\Omega_c}{\omega B_0} \Bigg\{ & \sum_{n=-\infty}^{+\infty} \frac{e^{i\beta \sin\varphi}\, e^{-in\varphi}}{\omega - k_\parallel v_\parallel - n\Omega_c} \left(\frac{n\Omega_c}{v_\perp} \frac{\partial f_0}{\partial v_\perp} + k_\parallel \frac{\partial f_0}{\partial v_\parallel} \right) \\ & \times \left[\frac{n \Omega_c}{k_\perp} J_n(\beta), \; i v_\perp J'_n(\beta), \; v_\parallel J_n(\beta) \right] \\ & + \\ & \left[\frac{\partial f_0}{\partial v_\perp} \cos\varphi, \frac{\partial f_0}{\partial v_\perp} \sin\varphi, \frac{\partial f_0}{\partial v_\parallel} \right] \Bigg\} \cdot \overline{E}_1 \end{aligned}}$$

avec $\beta = \dfrac{k_\perp v_\perp}{\Omega_c}$, f_0 symétrique, gyrotrope, $\Omega_c = \dfrac{q B_0}{M}$