1/

$$\partial_t f_s + \vec{v}\cdot\partial_{\vec{x}} f_s + \frac{q_s}{m_s}\left[\vec{E} + \vec{v}\times\vec{B}\right]\cdot\partial_{\vec{v}} f_s = 0$$

$$\vec{\nabla}\times\vec{E} = -\partial_t\vec{B} \qquad \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla}\wedge\vec{B} = \mu_0\vec{J} + \frac{1}{c^2}\partial_t\vec{E}$$

Vlasov à l'ordre 0 : $(\vec{v}\times\vec{B}_0)\cdot\partial_{\vec{v}} f_0 = 0$

$\rightarrow B_0(v_y\partial_{v_x} - v_x\partial_{v_y})f_0 = 0$ $\quad \left\{ \begin{array}{l} \partial v_x = \cos\theta\,\partial v_\perp - v_\perp\sin\theta\,\partial_\theta \\ \partial v_y = \sin\theta\,\partial v_\perp + v_\perp\cos\theta\,\partial_\theta \end{array}\right.$

$\hookrightarrow -B_0 v_\perp \partial_\theta f_0 = 0$

f_0 ne dépend que de $v_\parallel = v_z$ & $v_\perp$ (gyrotrope)

linéarisation à l'ordre 1 :

$$\partial_t f_1 + \vec{v}\cdot\partial_{\vec{r}} f_1 + \frac{q}{m}\vec{E}_1\cdot\partial_{\vec{v}} f_0 + \frac{q}{m}(\vec{v}\times\vec{B}_1)\partial_{\vec{v}} f_0 + \frac{q}{m}(\vec{v}\times\vec{B}_0)\cdot\partial_{\vec{v}} f_1 =$$

$\circledast \quad \circledast \quad \circledast$

$\rightarrow$ les 3 termes $\circledast$ sont égaux à $d_t f_1$ le long des orbites non perturbées : $d_t\vec{x} = \vec{v}$

$$d_t\vec{v} = \frac{q}{m}\vec{v}\times\vec{B}_0 .$$

$\leadsto d_t f_1 + \frac{q}{m}(\vec{E}_1 + \vec{v}\times\vec{B}_1)\cdot\partial_{\vec{v}} f_0 = 0$

de plus $\vec{\nabla}\times\vec{E}_1 = -\partial_t\vec{B}_1 \quad \rightarrow \quad \vec{B}_1 = \frac{\vec{k}\times\vec{E}_1}{\omega}$

d'où $\vec{v}\times\vec{B}_1 = \frac{\vec{v}}{\omega}\times(\vec{k}\times\vec{E}_1)$

$$= \frac{\vec{v}}{\omega}\cdot(\vec{E}_1\vec{k} - \vec{k}\vec{E}_1)$$

$$= \left(\frac{\vec{v}\cdot\vec{E}_1}{\omega}\right)\vec{k} - \left(\frac{\vec{k}\cdot\vec{v}}{\omega}\right)\vec{E}_1$$

$$\leadsto d_t f_1 + \frac{q}{m}\vec{E}_1\cdot\left(\vec{\vec{1}} + \frac{\vec{v}\vec{k}}{\omega} - \left(\frac{\vec{k}\cdot\vec{v}}{\omega}\right)\vec{\vec{1}}\right)\cdot\partial_{\vec{v}} f_0 = 0$$

avec $\vec{E}_1 = \vec{E}_1 e^{i(\vec{k}\vec{r} - \omega t)}$

soit $f_1 = -\frac{q}{m} \int_{-\infty}^{t} e^{i(\vec{k}.\vec{r}' - \omega t')} \vec{E}_1 . \left[\overrightarrow{\overrightarrow{1}} \left(1 - \frac{\vec{v}'.\vec{k}}{\omega}\right) + \frac{\vec{v}'\vec{k}}{\omega} \right] \partial_{\vec{v}} f_0 \, dt'$

où $r'(t')$, $v'(t')$ st les positions & vitesses de la part qui arrive en (r, v) à t le long de l'orbite non perturbée.

arg de l'exposant : $\vec{k} = k_{\parallel} \hat{z}$

$\vec{k}.\vec{r}' - \omega t' = \vec{k}.(z + v_{\parallel}\tau) - \omega t'$ avec $\tau = t' - t$

$= k v_{\parallel}(t' - t) - \omega t' + kz$

On cherche $f_1(r, v, t) = f_1(k, v, \omega)\, e^{i(\vec{k}\vec{r} - \omega t)}$.

alors $e^{i(\vec{k}\vec{r}' - \omega t')} = e^{i(k v_{\parallel}(t'-t) - \omega(t'-t))}\, e^{i(kz - \omega t)}$.

soit $f_1(k, v, \omega) = -\frac{q}{m} \int_{-\infty}^{t} e^{i(k v_{\parallel} - \omega)(t'-t)} dt' \, \vec{E}_1 . \left(\overrightarrow{\overrightarrow{1}} \left(1 - \frac{\vec{v}'.\vec{k}}{\omega}\right) + \frac{\vec{v}'\vec{k}}{\omega} \right) \partial_{\vec{v}} f_0$

Le dernier terme se réécrit :

$$T = \left(1 - \frac{v_{\parallel} k}{\omega}\right) \underbrace{\left(E_x \partial_{v_x} f_0 + E_y \partial_{v_y} f_0\right)}_{(*)} + \left(\vec{E}_1 . \vec{v}'_{\perp}\right) \frac{k}{\omega} \partial_{v_{\parallel}} f_0 .$$

$$\begin{cases} v'_x = v_{\perp} \cos\phi(t') \\ v'_y = v_{\perp} \sin\phi(t') \end{cases}$$

$(*) = E_x \partial_{v_x} f_0 + E_y \partial_{v_y} f_0$

$$\begin{cases} dv'_x = dv_{\perp} \cos\phi - v \sin\phi \, d\phi \\ dv'_y = dv_{\perp} \sin\phi + v \cos\phi \, d\phi \end{cases}$$

$$\hookrightarrow \begin{cases} dv_{\perp} = \cos\phi \, dv'_x + \sin\phi \, dv'_y \\ -v\, d\phi = \sin\phi \, dv'_x - \cos\phi \, dv'_y \end{cases}$$

puis
$$\begin{cases} \partial_{v_x} f = \cos\phi \, \partial_{v_{\perp}} f - \frac{1}{v} \sin\phi \, \partial_{\phi} f = \cos\phi \, \partial_{v_{\perp}} f_0 \\ \partial_{v_y} f = \sin\phi \, \partial_{v_{\perp}} f + \frac{1}{v} \cos\phi \, \partial_{\phi} f = \sin\phi \, \partial_{v_{\perp}} f_0 \end{cases}$$

$\cos\phi = \frac{v'_x}{v_{\perp}}$ $\quad \sin\phi = \frac{v'_y}{v_{\perp}}$

$\Rightarrow \vec{E} . \partial_{\vec{v}} f = E_x \cos\phi \, \partial_{v_{\perp}} f_0 + E_y \sin\phi \, \partial_{v_{\perp}} f_0$

$= \frac{\vec{E} . \vec{v}'_{\perp}}{v_{\perp}} \partial_{v_{\perp}} f_0$.

$$T = \left(1 - \frac{v_\parallel k}{\omega}\right)\left(\frac{\vec{E}_1 \cdot \vec{v}_\perp'}{v_\perp}\right)\partial_{v_\perp} f_0 + \left(\vec{E}_1 \cdot \vec{v}_\perp'\right)\frac{k}{\omega}\partial_{v_\parallel} f_0$$

$$\Rightarrow f_1(k,v,\omega) = -\frac{q}{m\omega}\int_{-\infty}^{0} e^{i(kv_\parallel - \omega)\tau}\, d\tau \left(\vec{E}_1 \cdot \vec{v}_\perp'\right)\left[k\partial_{v_\parallel} f_0 + \left(\frac{\omega - kv_\parallel}{v_\perp}\right)\partial_{v_\perp} f_0\right]$$

$$\tau = t' - t.$$

L'orbite non-perturbée satisfait à l'éq. :

$$\begin{cases} d_t \vec{x}' = \vec{v}' \\ d_t \vec{v}' = \Omega_c(\vec{v}' \times \hat{z}) \end{cases} \quad \text{avec} \quad \Omega_c = \frac{qB_0}{m}.$$

$$\vec{v}_\perp'(\tau) = v_+' \hat{e}_- + v_-' \hat{e}_+ \quad \text{avec} \quad \begin{cases} v_\pm = (v_x \pm i v_y)\frac{1}{\sqrt{2}} \\ \hat{e}_\pm = (\hat{x} \pm i\hat{y})\frac{1}{\sqrt{2}} \end{cases}$$

$$\rightarrow \begin{cases} v_z'(\tau) = v_\parallel \\ v_+'(\tau) = v_+ e^{-i\Omega_c \tau} \\ v_-'(\tau) = v_- e^{+i\Omega_c \tau} \end{cases}$$

Soit $\vec{E}_1 \cdot \vec{v}_\perp' = E_- v_+ e^{-i\Omega_c\tau} + E_+ v_- e^{+i\Omega_c t}$ (*)

$$\int_{-\infty}^{0} e^{i(kv_\parallel - \omega)\tau}\left(E_+ v_+ e^{-i\Omega_c\tau} + E_- v_- e^{+i\Omega_c\tau}\right)d\tau$$

$$= \frac{E_- v_+}{i(kv_\parallel - \omega - \Omega_c)} + \frac{E_+ v_-}{i(kv_\parallel - \omega + \Omega_c)}.$$

$$\rightarrow f_1(k,v,\omega) = \frac{-iq}{m\omega}\left[k\partial_{v_\parallel} f_0 + \left(\frac{\omega - kv_\parallel}{v_\perp}\right)\partial_{v_\perp} f_0\right]\left(\frac{E_- v_+}{kv_\parallel - \omega - \Omega_c} + \frac{E_+ v_-}{kv_\parallel - \omega + \Omega_c}\right)$$

Par ailleurs, en linéarisant maxwell :

$$\left(1 - \frac{k^2c^2}{\omega^2}\right)\vec{E} = \frac{-i}{\omega\varepsilon_0}\vec{J}$$

avec $\vec{J} = \sum_s q_s \int f_s \vec{v}\, d\vec{v}$ (ds l'espace (k,ω)) –

de plus, $\vec{k}\cdot\vec{E} = 0$ & $\vec{k} \parallel \vec{B_0}$ $\Rightarrow \vec{J} = \vec{J}_\perp$ –

(*) Δ $\left| \begin{array}{l} \hat{e}_+ \cdot \hat{e}_- = 1 \\ \hat{e}_+ \cdot \hat{e}_+ = \hat{e}_- \cdot \hat{e}_- = 0 \end{array} \right.$

On utilise toujours $\vec{v} = v_+ \hat{e}_- + v_- \hat{e}_+$

$$f_1 = \underbrace{\frac{iq}{m\omega}}_{A} \Big[\underbrace{k\partial_{v_\parallel} f_0 + \frac{(\omega - k v_\parallel)}{v_\perp} \partial_\perp f_0}_{B} \Big] \Big[\frac{v_+ E_-}{\underbrace{k v_\parallel - \omega - \Omega_c}_{C}} + \frac{v_- E_+}{\underbrace{k v_\parallel - \omega + \Omega_c}_{D}} \Big]$$

$$f_1 = AB \left(\frac{v_+ E_-}{C} + \frac{v_- E_+}{D} \right).$$

soit $A' = qA$.

$$\vec{J}_1 = q \int f_1 \vec{v}\, d^3v$$

$$= A' \int B \left(\frac{v_+^2 E_- \hat{e}_-}{C} + \frac{v_+ v_- E_+ \hat{e}_-}{D} + \frac{v_+ v_- E_- \hat{e}_+}{C} + \frac{v_-^2 E_+ \hat{e}_+}{D} \right) d^3v$$

$$\rightarrow \left\{ \begin{array}{l} \left(1 - \frac{k^2 c^2}{\omega^2}\right) E_+ = A' \int B \left(\frac{v_+^2 E_-}{C} + \frac{v_+ v_- E_+}{D} \right) d^3v \\ \left(1 - \frac{k^2 c^2}{\omega^2}\right) E_- = A' \int B \left(\frac{v_+ v_- E_-}{C} + \frac{v_-^2 E_+}{D} \right) d^3v \end{array} \right.$$

de la forme : $\alpha \begin{vmatrix} E_+ \\ E_- \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \cdot \begin{vmatrix} E_+ \\ E_- \end{vmatrix}$

$$\rightarrow \begin{pmatrix} \alpha - a & b \\ c & \alpha - d \end{pmatrix} \begin{pmatrix} E_+ \\ E_- \end{pmatrix} = 0.$$

7)

pour 1 onde polarisé E_+, 1 sol est $(E_- = 0)$.

$(\alpha - 2) = 0$

de m, pour 1 onde E_- $(E_+ = 0)$ est

$\alpha - d = 0$

de plus $v_+ v_- = \frac{v_\perp^2}{2}$

$$1 - \frac{k^2c^2}{\omega^2} = \frac{-i}{\omega\varepsilon_0} \sum_s \frac{i q_s^2}{m_s\omega} \int \left(k\,\partial_{v_\parallel} f_0 + \frac{(\omega - kv_\parallel)}{v_\perp}\partial_{v_\perp} f_0\right) \frac{v_\perp^2}{2} \frac{1}{\cancel{\omega_c} + kv_\parallel - \omega \mp \omega_c} d^3v$$

ie 2 eqs en $\mp$.

$$1 - \frac{k^2c^2}{\omega^2} - \sum_s \frac{q_s^2}{2m_s\varepsilon_0} \frac{1}{\omega^2} \int d^3v\, v_\perp \left(kv_\perp \partial_{v_\parallel} f_0 + (\omega - kv_\parallel)\partial_{v_\perp} f_0\right) \frac{1}{\omega - kv_\parallel \mp \Omega_{cs}} = 0.$$

avec $\tilde{f}_0 = \frac{F_0}{n_0}$

$$1 - \frac{k^2c^2}{\omega^2} - \sum_s \frac{\Omega_{ps}^2}{2\omega^2} \int d^3v\, v_\perp \left(kv_\perp \partial_{v_\parallel} F_0 + (\omega - kv_\parallel)\partial_{v_\perp} F_0\right) \frac{1}{\omega - kv_\parallel \mp \Omega_{cs}} = 0$$

$$I = \int d^3v \frac{v_\perp\left(kv_\perp \partial_{v_\parallel} F_0 + (\omega - kv_\parallel)\partial_{v_\perp} F_0\right)}{\omega - kv_\parallel \mp \Omega_{cs}}.$$

Pour 1 maxwellienne : $\alpha_\parallel = \frac{m}{2k_BT_\parallel}$ $\alpha_\perp = \frac{m}{2k_BT_\perp}$

$$F_0 = \left(\frac{\alpha_\parallel}{\pi}\right)^{1/2} \left(\frac{\alpha_\perp}{\pi}\right) e^{-\alpha_\parallel(v_\parallel - v_0)^2} e^{-\alpha_\perp v_\perp^2}.$$

$$\partial_{v_\parallel} F_0 = -2\alpha_\parallel (v_\parallel - V_0) F_0$$

$$\partial_{v_\perp} F_0 = -2\alpha_\perp v_\perp$$

soit $k v_\perp \partial_{v_\parallel} F_0 + (\omega - k v_\parallel)\partial_{v_\perp} F_0$

$$= -2k\alpha_\parallel v_\perp (v_\parallel - V_0) F_0 - 2\alpha_\perp (\omega - k v_\parallel) v_\perp F_0$$

$$= 2 v_\perp F_0 \left[k v_\parallel (\alpha_\perp - \alpha_\parallel) + k\alpha_\parallel V_0 - \omega \alpha_\perp \right]$$

d'où $I = 2\int d^3v \, \dfrac{v_\perp^2 F_0 \left[k v_\parallel (\alpha_\perp - \alpha_\parallel) + k\alpha_\parallel V_0 - \omega\alpha_\perp \right]}{\omega - k v_\parallel \mp \Omega_c}$

avec $A = 1 - \dfrac{T_\parallel}{T_\perp} \rightarrow \alpha_\parallel = \dfrac{1}{2v_{th}^2}$

et $v_{th}^2 = \dfrac{k_B T_\parallel}{m}$ $\quad \alpha_\perp = \dfrac{1-A}{2v_{th}^2}$

$$\frac{I}{2} = \int \frac{d^3v \, v_\perp^2 F_0}{\omega - k v_\parallel \mp \Omega_c} \left[\frac{k v_\parallel}{2v_{th}^2}(1 - A - 1) + \frac{kV_0}{2v_{th}^2} - \frac{\omega(1-A)}{2v_{th}^2} \right]$$

$$I = \int d^3v \, \frac{v_\perp^2 F_0}{v_{th}^2} \left(-A k v_\parallel + kV_0 - \omega(1-A) \right) \frac{1}{\omega - k v_\parallel \mp \Omega_c}$$

En faisant l'intégration $\int d^2v \, v_\perp^2 F_0$:

$$d^2v = 2\pi v_\perp dv_\perp \rightarrow \int 2\pi v_\perp^3 F_0 \, dv_\perp$$

$$= \cancel{2\pi} \frac{1}{\cancel{\alpha_\perp}} \frac{1}{2\alpha_\perp} \left(\frac{\alpha_\parallel}{\pi}\right)^{1/2} \cancel{\left(\frac{\alpha_\perp}{\pi}\right)} e^{-\alpha_\parallel (v_\parallel -}$$

$$= \frac{1}{\alpha_\perp} \left(\frac{\alpha_\parallel}{\pi}\right)^{1/2} e^{-\alpha_\parallel (v_\parallel - V_0)^2}$$

$$= \frac{1}{\alpha_\parallel (1-A)} \left(\frac{\alpha_\parallel}{\pi}\right)^{1/2} e^{-\alpha_\parallel (v_\parallel - V_0)^2}$$

$$= \frac{1}{1-A} \left(\frac{1}{\pi \alpha_\parallel}\right)^{1/2} e^{-\alpha_\parallel (v_\parallel - V_0)^2}$$

$$= \frac{1}{1-A} \left(\frac{2}{\pi}\right)^{1/2} v_{th} \, e^{-\frac{(v_\parallel - V_0)^2}{2v_{th}^2}}$$

$$I = \int \frac{dv_{\parallel}}{v_{th}\,\cancel{x}}\left[\frac{1}{1-A}\left(\frac{2}{\pi}\right)^{1/2} \cancel{v_{th}}\, e^{-\frac{(v_{\parallel}-v_0)^2}{2v_{th}^2}}\left(-Ahv_{\parallel}+hv_0-\frac{\omega(1-A)}{\cancel{2v_{th}^2}}\right)\right] \frac{1}{\omega - hv_{\parallel} \mp \Omega_c}$$

$$v = \frac{v_{\parallel}-v_0}{\sqrt{2}\,v_{th}} \qquad v^2 = \frac{(v_{\parallel}-v_0)^2}{2\,v_{th}^2}$$

$$dv = \frac{dv_{\parallel}}{\sqrt{2}\,v_{th}} \quad \to \quad v_{\parallel} = v\sqrt{2}\,v_{th} + v_0$$

$$I = \sqrt{2}\,\cancel{v_{th}}\,\frac{1}{\cancel{v_{th}}}\int dv\,\frac{1}{1-A}\left(\frac{2}{\pi}\right)^{1/2} e^{-v^2}\Big(-Ah(v\sqrt{2}\,v_{th}+v_0) + hv_0 - \omega(1-A)\Big)\frac{1}{\omega - h(v\sqrt{2}\,v_{th}+v_0) \mp \Omega_c}$$

$$I = \frac{2}{(\pi)^{1/2}}\int \frac{dv}{1-A}\,\frac{e^{-v^2}\left[-Ahv\sqrt{2}\,v_{th} + (hv_0-\omega)(1-A)\right]}{\sqrt{2}\,hv_{th}\left(\left(\frac{\omega - hv_0 \mp \Omega_c}{\sqrt{2}\,hv_{th}}\right) - v\right)}$$

$$I = \frac{2}{\sqrt{\pi}}\int \frac{dv}{1-A}\, e^{-v^2}\,\frac{\left[-Av + (1-A)(hv_0-\omega)/\sqrt{2}\,hv_{th}\right]}{\frac{\omega - hv_0 \mp \Omega_c}{\sqrt{2}\,hv_{th}} - v}$$

$$\xi = \frac{\omega - hv_0}{\sqrt{2}\,hv_{th}}$$

$$\xi^* = \frac{\omega - hv_0 \mp \Omega_c}{\sqrt{2}\,hv_{th}}$$

$$I = \frac{2}{\sqrt{\pi}}\int dv\; e^{-v^2}\frac{\left[-\frac{A}{1-A}v - \xi\right]}{\xi^* - v}$$

$$= \frac{2}{\sqrt{\pi}}\left(\frac{A}{1-A}\int\frac{e^{-v^2}\,v\,dv}{v-\xi^*} + \xi\int\frac{e^{-v^2}\,dv}{v-\xi^*}\right)$$

$$= \left[\frac{A}{1-A}\left(\frac{1}{2}\right)Z'(\xi^*) + \xi\, Z(\xi^*)\right]$$

soit $I = \frac{A}{1-A} Z'(\xi^*) + 2\xi Z(\xi^*)$

$\rightarrow$ idem résultat de Gary & Feldman 78

si $A = 1 - \frac{T_\perp}{T_\parallel}$

$\rightarrow I = A Z'(\xi^*) + 2\xi Z(\xi^*)$.

d'où l'éq. de dispersion :

$$1 - \frac{k^2c^2}{\omega^2} - \sum_s \frac{\Omega_{ps}^2}{2\omega^2}\left[A Z'(\xi^*) + 2\xi Z(\xi^*)\right] = 0$$

or $Z'(\xi) = -2\left[1 + \xi Z(\xi)\right]$

$$1 - \frac{k^2c^2}{\omega^2} - \sum_s \frac{\Omega_{ps}^2}{\omega^2}\left[-2A + 2Z(\xi^*)(\xi - A\xi^*)\right] = 0$$